Calculation of the Rate of $\Omega^- \rightarrow \Xi^0 + e^- + \bar{\nu}$*

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The rate of electronic decay of the Ω^- hyperon is calculated, making the assumptions of Cabibbo and using a Goldberger-Treiman relation for the axial vector part. The final result contains one free parameter.

WE consider here the electronic decay mode of the Ω^- hyperon.[1]

$$\Omega^- \rightarrow \Xi^0 + e^- + \bar{\nu}. \qquad (1)$$

It is assumed that this particle has spin-parity $3/2^+$ in conformity with the predictions of unitary symmetry.[2] Further, we assume $V-A$ leptonic coupling. The most general relativistic interaction is then

$$\bar{\psi}_\Xi\{(f_1+g_1\gamma_5)\delta_{\mu\lambda}+\gamma_\mu(f_2+g_2\gamma_5)q_\lambda+(f_3+g_3\gamma_5)q_\mu q_\lambda + (f_4+g_4\gamma_5)\sigma_{\mu\nu}q_\nu q_\lambda\}\psi_\Omega{}^\lambda\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu = M, \qquad (2)$$

where time reversal invariance implies that (f_i,g_i) are real functions of the momentum transfer squared, and we use the Rarita-Schwinger formalism[3] to describe the spin 3/2 hyperon. The expression for the rate, neglecting the lepton mass, and performing the angular integration, is

$$\frac{\Gamma}{2}=\Big\{2(r-1)^2J(f_1^2;1/2)-4(r-1/3)J(f_1^2;3/2) + (r-1)^2J(g_1^2;3/2)-2(r-1/3)J(g_1^2;5/2) + 4/3(r-1)\left[J(f_1f_2;3/2)\left(1+\frac{5r-1}{4}\right) + J(g_1g_2;3/2)\left(1-\frac{5r-1}{4}\right)\right] + 4J(f_2^2;5/2)+8J(g_2^2;3/2)\Big\}\sigma + \text{terms in } g_4 \text{ and } f_4 \cdots, \qquad (3)$$

where

$$J(h;n)=\int_0^{\epsilon_0}\epsilon^n d\epsilon h(\epsilon);\quad \epsilon=\frac{E_\Xi-M_\Xi}{M_\Xi},\quad \epsilon_0=\epsilon^{\max}\cong 0.03$$

$$r=M_\Omega/M_\Xi,\quad \sigma=\frac{\sqrt{2}M_\Xi{}^5G^2}{24(2\pi)^3}=1.58\times10^{11},\quad \text{sec}^{-1} \qquad (4)$$

and higher order terms in ϵ_0—$\cong 3\%$—have been neglected. The expression (3) is obtained using the projection operators given by Behrends and Fronsdal.[4] The form factors (f_3,g_3) do not appear as a result of neglecting the electron mass and using the Dirac equation. There are no interference terms of the form f_jg_i as a result of the symmetry properties of the lepton trace.[5]

Now we make the hypothesis, following Cabibbo,[6] that the vector part of the weak current—in this case the g_i terms in (3)—is in the same octet as the electromagnetic current. Further we adopt the definition of universality given in Ref. 6. Then we can relate $g_{1,2}(q^2=0)$ to the coupling constants in the photoproduction of N^* as follows:

$$g_{1,2,4}(0)=\sqrt{3}V_{1,2,4}\sin\theta, \qquad (5)$$

where the V's are defined[7] by the coupling to the photon:

$$[eV_1\bar{\psi}_N\gamma_5\psi_{N^*}{}^\mu+eV_2\bar{\psi}_N\gamma_\mu\gamma_5q_\nu\psi_{N^*}{}^\nu + eV_4\bar{\psi}_N\sigma_{\mu\nu}\gamma_5q_\nu q_\lambda\psi_{N^*}{}^\lambda]A_\mu \qquad (6)$$

and $\tan\theta$ is, as defined by Cabibbo,[6] just the factor that relates the $\Delta S=0$, and $\Delta S=1$ weak currents while the factor $\sqrt{3}$ comes from the Clebsch-Gordan coefficients for SU_3.[8] From Refs. 6 and 7, respectively, we get

$$\sin\theta=0.26\quad V_1=(M_{N^*}+M_N)V_2;\quad V_2=0.37/M_\pi;\quad V_4=0. \qquad (7)$$

We now evaluate the contribution of the axial current. Assuming the matrix element of the axial current to be dominated by the K^- pole, we obtain a relation of the Goldberger-Treiman type[9]:

$$[f_1+(M_\Omega-M_\Xi)f_2]_{q^2=0}=\gamma_{\Omega\Xi K}F/M_K, \qquad (8)$$

where the f_4 piece does not appear since its divergence is zero. Again using SU_3 invariance, we get

$$\gamma_{\Omega\Xi K}=\sqrt{3}\gamma_{N^*N\pi} \qquad (9)$$

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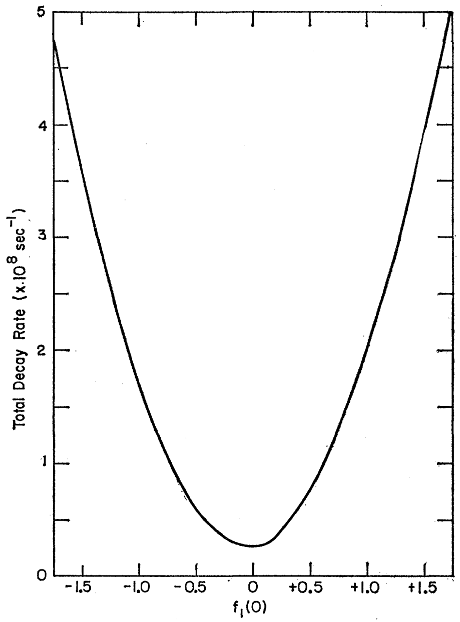

FIG. 1. The total decay rate is shown as a function of the dimensionless quantity $f_1(0)$.

F is related to the rate of the decay $K^- \rightarrow \mu^- + \bar{\nu}$ by

$$\Gamma_{K^- \rightarrow \mu^- + \bar{\nu}} = \left(\frac{m_\mu}{m_K}\right)^2 \frac{F^2}{M_K} \frac{G^2(M_K^2 - M_\mu^2)^2 \sin^2\theta}{8\pi}. \quad (10)$$

Using the experimental value[10] $\tau_{K^-} = 1.22 \times 10^{-8}$ sec, and the branching ratio[10] 64%, we get

$$\sin\theta F/M_K = 5.88 \times 10^{-2}. \quad (11)$$

From Ref. 7 we get $\gamma_{N^*N\pi} = 2.07$. Substituting this in (8) and using (11),

$$f_1(0) + (M_\Omega - M_\Xi) f_2(0) = 0.208. \quad (12)$$

Assuming the form factors to be constant over the physical range of q^2 and setting them equal to their values at $q^2 = 0$, we get for the vector and axial vector contributions to the rate:

$$\Gamma_V = 2.45 \times 10^7 \text{ sec}^{-1};$$

$$\Gamma_A = [1.53 f_1^2(0) + 0.15 f_1(0) + 0.075] \times 10^8 \text{ sec}^{-1}. \quad (13)$$

In Fig. 1 the total decay rate, $\Gamma = \Gamma_A + \Gamma_V$, is plotted as a function of $f_1(0)$.

If we use $\tau_{\Omega \rightarrow \Xi^0 \pi^-} = 0.7 \times 10^{-10}$ sec and $\tau_{\Omega \rightarrow K\Lambda} = 1.3 \times 10^{-10}$ sec,[11] we get an approximate lower limit to the branching ratio:

$$\frac{\Gamma_{\Omega \rightarrow \Xi^0 e^- \bar{\nu}}}{\Gamma_{\text{total}}} \gtrsim 0.54\%. \quad (15)$$

Note added in proof: Recently, a preprint was received on this subject from Professor J. Mathews. His calculation gives $V_2 = 0.26/m_\pi$ as opposed to $0.37/m_\pi$ as in (7). The vector rate then becomes $\Gamma_V = 1.25 \times 10^7$ sec^{-1}, according to (3). Mathews' calculation gives $\Gamma_V = 1.07 \times 10^7$ sec^{-1}, making no approximation to the integrals and including the lepton mass.

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